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# Mathematical Modeling of Nigeria's Gross Domestic Product with Focus on its Economic Status

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**The optimal investment's decision is the function of the strategy adopted for creative opportunities that is arbitrage free, subject to guaranteed liquidity. This decision is subject to value characterization of evolving economy linked to annual fluctuation of the real Gross Domestic Product (GDP). GDP is an indicator for financial analysts, investors, and decision makers to abdicate an economy as contracting or expanding. In most developing countries (particular in Nigeria), GDP is affected by: Consumption, Export, Imports, Investments, Government Expenditure, Tax, and related corrupt sharp practices. We considered an extension of the models formulated by Penrose, Alex, Viktor and Sirhiy by accommodating noise to each component of the GDP model. This led to mathematical formulation of a vector stochastic differential equation for describing Nigeria GDP. An analytic solution was obtained using Ito formula. Mathematical formulations for drift and volatility vectors were obtained. Necessary data was obtained from the official website of the Government of Nigeria. The value for historical volatility calculated, $\|\underline{\sigma}\|_2$ is 0.356877114, and drift, $\|\underline{\alpha}\|_2$ is 0.342842212. Thus, $\|\underline{\sigma}\|_2 > \|\underline{\alpha}\|_2$, and Nigeria economy experiences inflation between first quarter and third quarter of 2022.**

**Math. Subj. Classification 2010**: 28A10, 52A20, 60G15, 60G17, 90B30, 91G10

**Key words:** Gross Domestic Product, Positive Definite, Standard Wiener Processes, Topological Linear Space, Vector Stochastic Differential Equation

## INTRODUCTION

Penrose (2010); Alex (2013); Viktor and Sirhiy (2019), Kitov (2006); Raja and Amalina (2017) expressed GDP of a geographical entity as linear combination of consumption, export, import, investment, government expenditure and tax without consideration for noise associated with each component that summed to the defined GDP.

Thus, the motivation to consider mathematical expression that extends their model by accommodating environmental noise base on the principle of vector and stochastic analysis, and vector topology. More information is available in: Cowper (1926, reprinted 1956); Ioannis and Steven (1998, 1991) for stochastic analysis; Bogachev and Smolyanov (2017) for analysis on topological vector space. This enables an intending investor to measure true nature of the economy, and hence pronounce the economy as either expanding or contracting. See Rubinfeld and Pindyck (2013) and the references there in with respect to microeconomic theory.

The study considers the sets; Ω (of all recorded activities in Nigeria economy), $\beta$ (of available economy related data that forms a net of sequence), and the measure μ on the measurable space (Ω, $\beta$) to capture

the existing Nigeria macro economy activities from 2010 till 2019 such that:

1. $\mu(\Omega) = 1$ almost surely.
2. $\mu(A) \geq 0$ for any $A \in \Omega$
3. $\mu(UjAj) = \Sigma j\, \mu(Aj) = 1$

## MATHEMATICAL MODEL OF THE GDP

Economist expert describes GDP as a gauge (in terms of economic activity) of the net total output produced in an economy.

Consider a random variable G defined on a filtered probability space $(\Omega, \beta, F(\beta), \mu)$ such that

$$G: [0, \infty) \times \Omega \rightarrow \mathbb{R}^n$$

with

$G^{-1}(A) = \{\omega \in \Omega: G(\omega) \in A\} \in \beta_t$

for each non-empty measurable set, $A \in B(\mathbb{R}^n)$, $n \in$ N; $1 < n < \infty$ and $\beta t \in F(\beta)$.

The dynamics of G satisfies the Vector Stochastic Differential Equation (VSDE):

$$\frac{dG(t,\omega)}{G(t,\omega)} = \underline{\alpha} dt + \underline{\sigma} dW(t,\omega), \quad G(t_\circ, \omega) = G_\circ \tag{1}$$

where;

$$G(t,\omega) = \left(a_{i,j}(t,\omega)\right)_n \tag{2}$$

$$a_{i,j}(t,\omega) = \begin{cases} a_{i,i}(t,\omega), & if\ i = j \\ 0, & if\ i \neq j \end{cases} \quad i,j = 1, 2. \cdots, n \tag{3}$$

$$\underline{\alpha}(t) = \left(e_j\right)_n \left(\alpha_j(t)\right)_n^T$$

$$\underline{\sigma}(t) = \left(e_j\right)_n \left(\sigma_j(t)\right)_n^T \tag{4}$$

$$j = 1, 2, \cdots, n$$

are real-valued functions of a real variable t, and $e_j$ is the natural basis of $\mathbb{R}^n$ for each j.

$\alpha_j(t)$ and $\sigma_j(t)$ for each j are at least once differentiable with respect to time, t.

$$dW(t,\omega) = \left(dw_{i,j}(t,\omega)\right)_n \quad i,j = 1, 2, \cdots, n \tag{5}$$

$$dw_{i,j}(t,\omega) = \begin{cases} dw_{i,j}(t,\omega), & if\ i = j \\ 0, & if\ i \neq j \end{cases} \tag{6}$$

- $w_{i,j}$ are the standard Wiener processes of n-dimension with respect to the $\sigma$-algebra.
- $\underline{\alpha}$ is the appreciated and adapted vector rate such that $G(t_\circ, \omega) \neq 0$, $t \in [t_\circ, \infty)$.
- $\underline{\sigma}$ measures the selectivity of the economy noise (that is, the change in a random number within variety of intervals) associated with the macro-economy variables**.** See Fanelli (2008); Reinhold and Bernd (2005) and the references there in for illustrations on volatility.

The drift vector and volatility vector of (1) satisfy the condition.

$$\int_{t_\circ}^{\infty} \left(\|\underline{\alpha}\|_2 + \|\underline{\sigma}\|_2\right) dt < \infty$$

and

$$\mu(G(t_\circ, \omega) = G_\circ) = 1$$

The integral expression for (1) is:

$$G(t,\omega) = G(t_\circ, \omega) + \int_{t_\circ}^{t} \underline{\alpha} G(s,\omega)\, ds + \int_{t_\circ}^{t} \underline{\sigma} G(s,\omega)\, dW(s,\omega) \tag{7}$$

Let

$$f: [0, \infty) \times \mathbb{R}^n \longrightarrow \mathbb{R}$$

with continuous partial derivatives

$$\frac{\partial f}{\partial t}, \quad \frac{\partial f}{\partial G_i(t,\omega)}, \quad \frac{\partial^2 f}{\partial G_i(t,\omega) \partial G_j(t,\omega)}$$

for $i, j = 1, 2, \cdots, n$; the process $\{X(t), t \in [0, \infty)\}$ defined by

$$X(t) = f(t, G(t, \omega)) \tag{8}$$

with probability one, where $G(t, \omega)$ satisfies the SDE (1). Then, by Ito definition, we have

$$dX(t) = \left\{\frac{\partial f}{\partial t} + \underline{\alpha}\nabla f + \frac{1}{2}\underline{\sigma}\,\underline{\sigma}^T tr(\nabla^2 f)\right\} dt + \underline{\sigma}\nabla f^T dW(t, \omega) \tag{9}$$

in vector-matrix notation.

**SOLUTION OF THE VSDE**

Let

$$f(t, G(t, \omega)) = lnG(t, \omega) \tag{10}$$

where $G(t, \omega)$ satisfies (1).

Thus, we have;

$$df(t, G(t, \omega)) = d(ln(t, G(t, \omega)) - \frac{1}{2}\frac{d < G(t, \omega) >}{G^2(t)} \tag{11}$$

for a fixed $\omega \in \Omega$.

But

$$< G(t, \omega) > = |\alpha| G^2(t, \omega) dt \tag{12}$$

of the quadratic variation of G, so that

$$df\big(t, G(t, \omega)\big) = \left(\underline{\alpha} - \frac{1}{2}|\underline{\sigma}|\right) dt + \underline{\sigma} dW(t, \omega) \tag{13}$$

Integrating (13) from $t_\circ$ to t for $t > t_\circ$, $t_\circ, t \in [0, \infty)$, we have;

$$f(t, G(t, \omega)) - f(t_\circ, G(t_\circ, \omega)) = (\underline{\alpha} - \frac{1}{2}|\underline{\sigma}|)(t - t_\circ) + \underline{\sigma}(W(t, \omega) - W(t_\circ, \omega)) \tag{14}$$

$$ln\left(\frac{f(t, G(t, \omega))}{f(t_\circ, G(t_\circ, \omega))}\right) = (\underline{\alpha} - \frac{1}{2}|\underline{\sigma}|)(t - t_\circ) + \underline{\sigma}(W(t, \omega) - W(t_\circ, \omega)) \tag{15}$$

$$G(t, \omega)) = G(t_\circ, \omega)) \exp\left((\underline{\alpha} - \frac{1}{2}|\underline{\sigma}|)(t - t_\circ) + \underline{\sigma}(W(t, \omega) - W(t_\circ, \omega))\right) \tag{16}$$

$$= G(t_\circ, \omega)) \exp\left((\underline{\alpha} - \frac{1}{2}|\underline{\sigma}|)(t - t_\circ) + \underline{\sigma}(W(t, \omega))\right) \tag{17}$$

where in (17);

- $G(t, \omega)$ satisfies (2) and (3)
- $\underline{\alpha}$ *and* $\underline{\sigma}$ satisfies (4). $|\underline{\sigma}|$ is a scalar.
- *dW*(*t, ω*) satisfies (5) and satisfies (6)

for $W(t_\circ = 0, \omega) = 0$.

Let $\lambda_j$ denote the eigenvalue of the matrix $G(t, \omega)$ in (17) for *j* = 1,2,⋯ ,*n.* Then, we express (17) as;

$$\mathcal{G}(t, \omega) = \Bbbk G(t_\circ, \omega) \exp\left((\|\underline{\alpha}\| - \frac{1}{2}|\underline{\sigma}|)(t - t_\circ) + \|\underline{\sigma}\|(W(t, \omega))\right) \tag{18}$$

$$0 \leq t_0 < t \leq t_* < \infty$$

where

$$\Bbbk = \begin{cases} \mathrm{a}, & \text{if } \sum_{\mathrm{j}=1}^{\mathrm{n}} \lambda_{\mathrm{j}} \geq 0 \\ -\mathrm{a}, & \text{if } \sum_{\mathrm{j}=1}^{\mathrm{n}} \lambda_{\mathrm{j}} < 0 \end{cases} \tag{19}$$

**REMARK**

Equation (18) is the gauge expression of the total output for all economic activities under study.

**THEOREM 1**

*Consider the vector space* $\mathbb{R}^n(\mathbb{R})$ *and the vectors* $\underline{\alpha}, \underline{\sigma} \in \mathbb{R}^n(\mathbb{R})$ *such that* $\cup_{\underline{\alpha}}$ *and* $\cup_{\underline{\sigma}}$ *are respective neighborhood of* $\underline{\alpha}$ *and* $\underline{\sigma}$*. If for* $\underline{v} = \underline{\alpha} + \underline{\sigma}$*, there exist* $\cup_{\underline{v}}$ *such that* $\cup_{\underline{\alpha}} + \cup_{\sigma} \subset \cup_{v}$*, and* $s \in \mathbb{R}$*,* $s\underline{\alpha} \in \mathbb{R}^n$ *with* $\cup_s = \{z : |z - s| < r, r > 0\}$*. Thus, vector addition and multiplication are continuous.*

**DEFINITION**

1. Consider the linear space $\mathbb{R}^n$ endowed with a topology in which the operations of vector addition and vector multiplication by scalars are continuous. Then, the space is called a topological vector space.
2. Let $\{\underline{\alpha}_n\}$ denote sequence of points in a topological linear space. If for any neighborhood of zero contain all the points $\|\underline{\alpha}_n - \underline{\alpha}_m\|$ at $n, m > N$, $N \in \mathbb{N}$, then the linear space is called fundamental.
3. The sequence in a fundamental topological linear space is Cauchy.
4. A topological linear space is complete if any Cauchy sequence has the limit in it.

**THEOREM 2**

Consider the dynamics:

$$\frac{dG(t,\omega)}{G(t,\omega)} = \underline{\alpha}dt + \underline{\sigma}dW(t,\omega) \qquad G(t_\circ, \omega) = G_\circ$$

where;

$$G(t,\omega) = \left(a_{i,j}(t,\omega)\right)_n$$

associated with the macro-economy variables defined on a filtered probability space, ($\Omega$, $\beta$, $F(\beta)$, $\mu$) with respect to an economy; the vectors $\underline{\alpha}$, $\underline{\sigma}$ are elements of a complete linear topological space V endowed with Euclidean norm, and the solution of the dynamics.

$$\mathcal{G}(t,\omega) = \Bbbk G(t_\circ,\omega)\exp\left((\|\underline{\alpha}\| - \frac{1}{2}|\underline{\sigma}|)(t - t_\circ) + \|\underline{\sigma}\|(W(t,\omega))\right)$$

for $0 \leq t_{\square} < t \leq t_* < \infty$, where $\Bbbk$ is the eigenvalue property of $G(t, \omega)$.
If;

- $\Bbbk > 0$ and $\|\underline{\alpha}\| > \|\underline{\sigma}\|$, then, the sample path of $\mathcal{G}(t,\omega), \omega \in \Omega$ for $0 \leq t_{\square} < t \leq t_* < \infty$ varies continuously slowly positively.
- $\Bbbk > 0$ and $\|\underline{\alpha}\| < \|\underline{\sigma}\|$, then, the sample path of $\mathcal{G}(t,\omega), \omega \in \Omega$ for $0 \leq t_{\square} < t \leq t_* < \infty$ varies continuously rapidly positively.
- $\Bbbk < 0$ and $\|\underline{\alpha}\| > \|\underline{\sigma}\|$, then, the sample path of $\mathcal{G}(t,\omega), \omega \in \Omega$ for $0 \leq t_{\square} < t \leq t_* < \infty$ varies continuously slowly negatively.
- $\Bbbk < 0$ and $\|\underline{\alpha}\| < \|\underline{\sigma}\|$, then, the sample path of $\mathcal{G}(t,\omega), \omega \in \Omega$ for $0 \leq t_{\square} < t \leq t_* < \infty$ varies continuously rapidly negatively.

**REMARK 2.1**

1. Let $(t_k, t_{k+1}]$ denotes a quarter in an economy, where $t_k = 3k$, $t_{k+1} = 3(k + 1)$, $k \in \mathbb{Z}^+$.
2. If the sample path of $\mathcal{G}(t,\omega), \omega \in \Omega$ varies continuously negatively for $t_k < t_* \leq t_{k+1}$, $t_{k+1} = 6(k + 1)$, the economy is in recession.
3. If the sample path of $\mathcal{G}(t,\omega), \omega \in \Omega$ varies continuously positively for $t_k < t_* \leq t_{k+1}$, $t_{k+1} = 6(k + 1)$, the economy is in inflation.

**PROOF OF THEOREM 2**

Consider the Complete topological linear space V equip with a topology build from Euclidean norm. Let $\mathcal{E}$ denotes a base of neighborhood of zero for the topology. Then, the elements of $\{B_{\min\{ri\}}(0) : \| \underline{x} \| \leq \min\{r_i\}, r_i > 0, i = 1,2\}$ are closed circled set, and for every $s\underline{\alpha}, s\underline{\sigma} \in \mathcal{E}$, $s_1, s_2 \in \mathbb{R}$, $s_1 s_2 \neq 0$, $\underline{\alpha}, \underline{\sigma} \in B_{\min\{ri\}}(0)$; we have the line segment $s_1\underline{\alpha} + s_2\underline{\sigma} \in \mathcal{E}$. If $\| \underline{\sigma} \| > \| \underline{\alpha} \|$, the random variable $G(t, \omega)$ varies continuously rapidly, but varies continuously slowly provided $\| \underline{\sigma} \| < \| \underline{\alpha} \|$.
Let a: = sum of the principal minor of the eigen value of the matrix $G(t, \omega)$. Then, if;

1. $a > 0$, and
i $\| \underline{\sigma} \| > \| \underline{\alpha} \|$, then, the sample path of $\mathcal{G}(t, \omega), \omega \in \Omega$ is continuously rapidly positively, else
ii $\| \underline{\sigma} \| < \| \underline{\alpha} \|$, the sample path of $\mathcal{G}(t, \omega), \omega \in \Omega$ is continuously slowly positively.
2) $a < 0$
i $\| \underline{\sigma} \| > \| \underline{\alpha} \|$, then, the sample path of $\mathcal{G}(t, \omega), \omega \in \Omega$ is continuously rapidly negatively, else
ii $\| \underline{\sigma} \| < \| \underline{\alpha} \|$, the sample path of $\mathcal{G}(t, \omega), \omega \in \Omega$ is continuously slowly negatively.

Hence, the economy is in recession if the sample path of $\mathcal{G}(t, \omega), \omega \in \Omega$ varies continuously negative for $t \in (t_k, t_{k+1}]$, $t_k = 6k$, $t_{k+1} = 6(k + 1)$, $k \in \mathbb{Z}^+$. Otherwise, the economy is in inflation if the sample path of $\mathcal{G}(t, \omega), \omega \in \Omega$ varies continuously positive for $t \in (t_k, t_{k+1}]$, $t_k = 6k$, $t_{k+1} = 6(k + 1)$, $k \in \mathbb{Z}^+$.

**HISTORICAL DERIVATION OF $\underline{\alpha}$(t)**

Consider the set of underlying

$$\mathbf{H} = (h_{ij})_{t,n} \tag{20}$$

constituting the macro-economic activities of the economy, Ω under study, (Adachi, (2020)) for more illustrations on multivariate data analysis.
Here, $(h_{1,j}, h_{2,j}, h_{3,j}, \cdots, h_{t,j})^T$ is the record of $j^{th}$ underlying's contribution to the economy for year $i \in [1, t]$, $t < \infty$. Thus,

$$\alpha_j(t) = \frac{1}{t} 1_t h_{i,j} \tag{21}$$

where, $1_t = (1,1,1,\cdots,1)$ is the average weight contribution of underlying j in the macro-economy. The drift vector is

$$\underline{\alpha}(t) = \left(\alpha_1(t), \alpha_2(t), \alpha_3(t), \cdots, \alpha_n(t)\right) \tag{22}$$

**HISTORICAL DERIVATION OF $\underline{\sigma}$(t)**

The deviation around the $j^{th}$ underlying with respect to the macro-economy for the period of i years, $i \in [1, t]$ is express by;

$$\mathrm{I_t} h_{i,j} - \tfrac{1}{t} h_{i,j} = \left(\mathrm{I_t} - \tfrac{1}{\mathrm{t}} \mathrm{I_t} \mathrm{I_t^T}\right) h_{i,j} = \mathfrak{I} h_{i,j} \tag{23}$$

where $\mathrm{I_t}$ is an identity matrix of order $t \times t$, and $\mathcal{J} = I_t - \frac{1}{t} 1_t 1_t^T$

**REMARK**

- $\mathfrak{I} = \mathrm{I_t} - \frac{1}{\mathrm{t}} \mathrm{I_t} \mathrm{I_t^T}$ (24)
- $= I_t - 1$ (25)
- but $\mathfrak{I} = I_t - 1 = \mathfrak{I}^T$ (26)
- Thus, $\mathfrak{I}$ is symmetric, and $I_t \mathfrak{I} = I_t \left(I_t - \frac{1}{\mathrm{t}} \mathrm{I_t} \mathrm{I_t^T}\right) = 0$
- $\mathfrak{I}\mathfrak{I} = \mathfrak{I}^T = \mathfrak{I}$ so that $\mathfrak{I}$ is idempotent.

The variance of the $j^{th}$ underlying with respect to the macro-economy is expressed by;

$$\mathfrak{I} h_{i,j} = \tfrac{1}{t} \left(\mathfrak{I} h_{i,j}\right)^T \mathfrak{I}^T \mathfrak{I} h_{i,j} = \tfrac{1}{t} \left(h_{i,j}\right)^T \mathfrak{I} h_{i,j} \tag{27}$$

**REMARK**

- Equation (27) is a quadratic variation of the vector ($h_{1,j}, h_{2,j}, h_{3,j}, \cdots, h_{t,j}$)
- Let $\frac{1}{t}\left(h_{i,j}\right)^T \Im h_{i,j}$ be positive definite, then the principal minors associated with $\frac{1}{t}\left(h_{i,j}\right)^T \Im h_{i,j}$ are all non-negative.

Thus, the standard deviation of the $j^{th}$ underlying for i years, $i \in [1, t]$ with respect to the macro-economy is expressed by;

$$\sigma_j(t) = \sqrt{\frac{1}{t}\left(h_{i,j}\right)^T \Im h_{i,j}} \tag{28}$$

The volatility vector is $\underline{\sigma}(t) = \left(\sigma_1(t), \sigma_2(t), \sigma_3(t), \cdots, \sigma_n(t)\right)$

**THE NIGERIA GDP**

In this section, we consider an investor that intends to understand the direction (status) of an economy before making investment decision.

**PROBLEM STATEMENT**

The investor desire to determine if the economy of interest is contracting or expanding.

**PROBLEM FORMULATION**

We define the decision variable as follows:
Let:

- $(h_{1,j}, h_{2,j}, h_{3,j}, \cdots, h_{t,j})^T$ denote the record contribution of the $j^{th}$ underlying with respect to the Nigeria macro-economic activities, $j = 1, 2, \cdots, 22$.
- $\left(\alpha_1(t), \alpha_2(t), \alpha_3(t), \cdots, \alpha_{22}(t)\right)$ denote the drift vector with respect to the records of the 22 underlying in the Nigeria macro-economy.
- $\left(\sigma_1(t), \sigma_2(t), \sigma_3(t), \cdots, \sigma_{22}(t)\right)$ denote the volatility vector with respect to the records of the 22 underlying in the Nigeria macro-economy.

What is the nature of Nigeria economy base on the recorded activities?
The status of Nigeria economy is obtained by:

$$\mathcal{G}(t,\omega) = \Bbbk G(t_\circ, \omega) \exp\left( (\|\underline{\alpha}\| - \frac{1}{2}|\underline{\sigma}|)(t - t_\circ) + \|\underline{\sigma}\|(W(t,\omega)) \right)$$

for $0 \leq t_\circ < t < \infty$, where k is the eigenvalue property of $G(t, \omega)$. The numerical values for each of $\alpha_j(t)$ and $\sigma_j(t)$ were calculated using equations (20) and (28) respectively.
We used Microsoft-Excel package to evaluate the mathematical formulations.

**ANALYSIS OF RESULT**

The value of $\|\underline{\sigma}\|_2$ is 0.356877114 by the readings on the chart of historical volatility (Figure 2 in the Appendix), and that of $\|\underline{\alpha}\|_2$ is 0.342842212 from the readings on chart of historical drift (Figure 3 in the Appendix).

Thus, $\|\underline{\sigma}\|_2 > \|\underline{\alpha}\|_2$, the sample path of $\mathcal{G}(t,\omega), \omega \in \Omega$ varies continuously positive between the first quarter to the third quarter of 2022 by the chart of Nigeria GDP (Figure 1). Hence, by the statement of the remark (2.1) to the Theorem (2), the Economy of Nigeria experienced inflation within the period of first quarter to the third quarter of 2022 all things being equal. This trend was expected to extend to fourth quarter of 2022. Most of the underlying by the graph displayed on Figure 4 of the Appendix in the macro-economy understudied are volatile. Nigeria economy is expected to shrink by first quarter of 2023 but expand by

second quarter of 2023 all things being equal. Hence, the need for the investor to optimize his proposition critically for optimal investment decision.

**Figure 1.** Stimulated sample path of the random variable, $\mathcal{G}(t,\omega), \omega \in \Omega$ with respect Nigeria GDP from 2010 to 2023.

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## APPENDIX

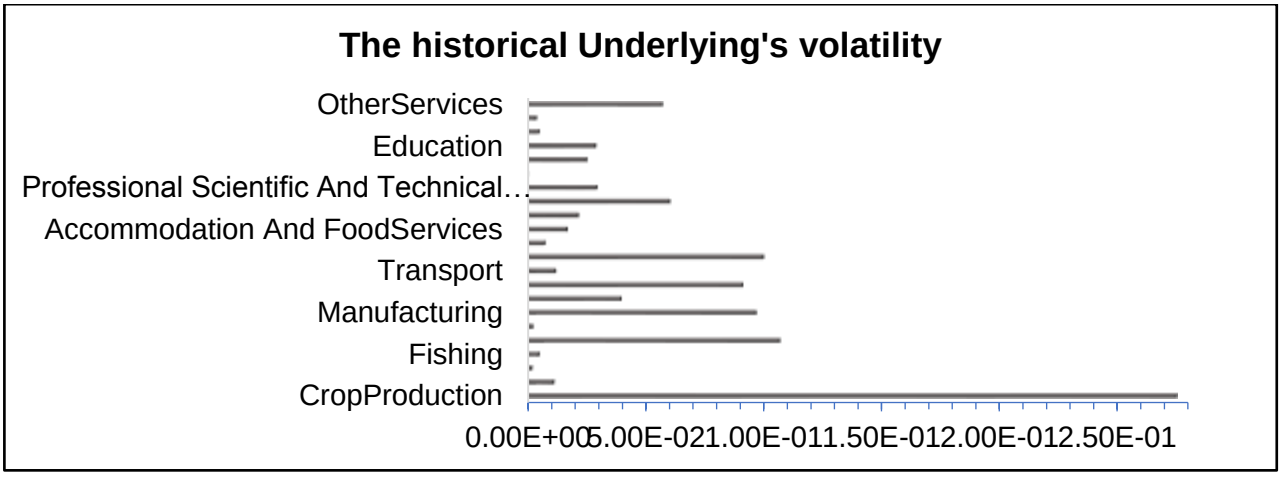


**Figure 2.** Chart of historical volatility with respect Nigeria GDP data from 2010 to 2019.

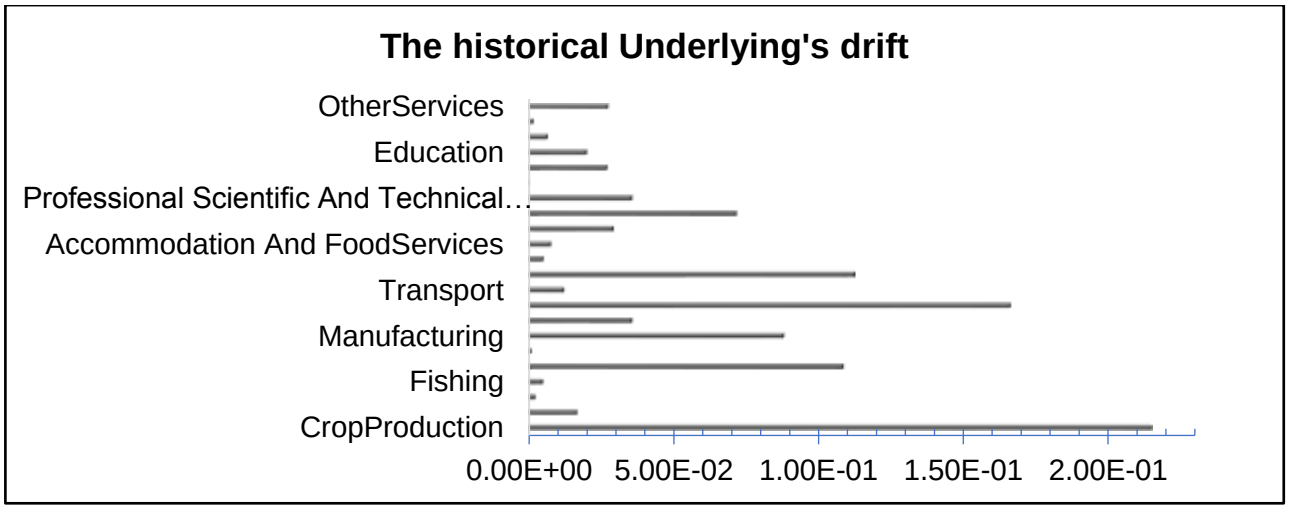


**Figure 3.** Chart of historical drift with respect Nigeria GDP data from 2010 to 2019.

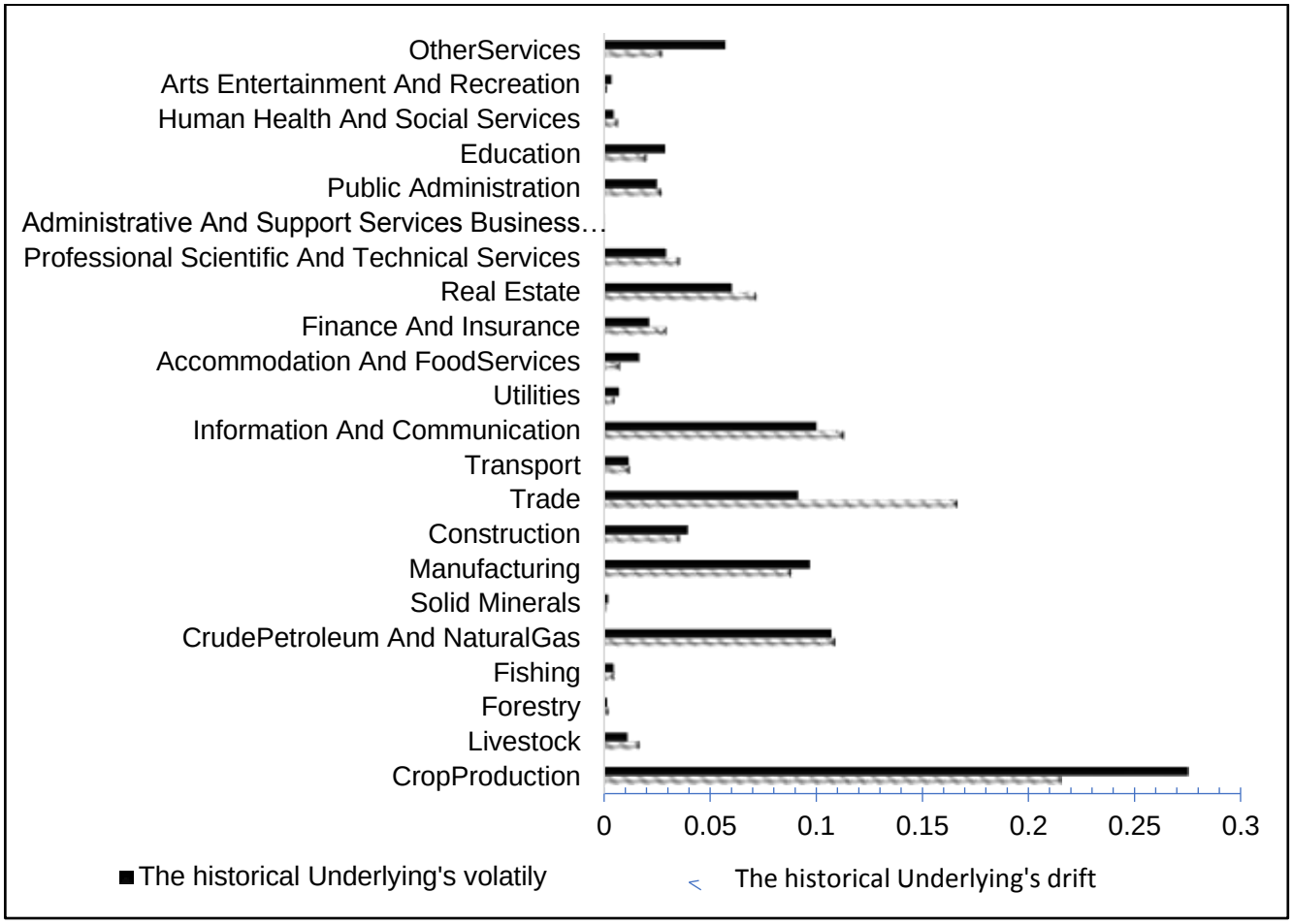


**Figure 4.** Chart of historical drift and volatility with respect to Nigeria GDP data from 2010 to 2019.